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## $|V_{ub}|$ from exclusive $B$ and $D$ decays <sup>\*</sup>

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### Abstract

We propose a model-independent method to determine the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element  $|V_{ub}|$  from exclusive  $B$  and  $D$  decays. Combining information obtainable from  $B \rightarrow \rho \ell \bar{\nu}_\ell$ ,  $B \rightarrow K^* \nu \bar{\nu}$ ,  $D \rightarrow \rho \bar{\ell} \nu_\ell$  and  $D \rightarrow K^* \bar{\ell} \nu_\ell$ , a determination of  $|V_{ub}|$  is possible, with an uncertainty from theory of around 10%. Theoretical uncertainties in the  $B \rightarrow K^* \ell \bar{\ell}$  decay rate are discussed.

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## I. INTRODUCTION

In the minimal standard model the couplings of the  $W$ -bosons to the quarks are given in terms of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix,  $V_{ij}$ , which arises from diagonalizing the quark mass matrices. In the minimal standard model (*i.e.*, one Higgs doublet) it is this matrix that is responsible for the  $CP$  nonconservation observed in kaon decay. A precise determination of the elements of the CKM matrix will play an important role in testing this picture for the origin of  $CP$  violation, and will constrain extensions of the standard model that make predictions for the form of the quark mass matrices.

The present value of the  $b \rightarrow u$  element of the CKM matrix,  $|V_{ub}| \simeq (0.002 - 0.005)$  [1] arises from a comparison of the endpoint region of the electron spectrum in semileptonic  $B$  decay with phenomenological models. In recent years there has been a dramatic improvement in our understanding of the theory of inclusive semileptonic  $B$  decays [2–4]. It was shown that the electron energy spectrum,  $d\Gamma/dE_e$ , can be predicted including nonperturbative strong interaction effects that are parameterized by the matrix elements of local operators between  $B$  meson states. For typical values of the electron energy  $E_e$ , the lowest dimension operators are the most important and the small nonperturbative strong interaction corrections are dominated by only two matrix elements, one of which is already determined by the measured  $B^* - B$  mass splitting [3,4]. However, for the semileptonic decay rate in the endpoint region,  $(m_B^2 - m_D^2)/2m_B < E_e < (m_B^2 - m_\pi^2)/2m_B$ , (where low mass hadronic final states are more important) the nonperturbative strong interaction corrections are large and an infinite set of nonperturbative matrix elements are needed. It has been shown that the same matrix elements determine the rate for  $B \rightarrow X_s \gamma$  in the region where the photon energy is near its maximal value [5]. In principle, experimental information on  $B \rightarrow X_s \gamma$  can be used to predict the electron spectrum in the endpoint region of semileptonic  $B$  decay, leading to a model-independent determination of  $|V_{ub}|$ .

In this paper we propose a method for getting a precise model-independent value for  $|V_{ub}|$  using exclusive  $B$  and  $D$  decays. Our approach gives a value of  $|V_{ub}|$  that (apart

from some very small factors) is valid in the limit of  $SU(3)$  flavor symmetry (on the  $u$ ,  $d$  and  $s$  quarks) or in the limit of  $SU(4)$  heavy quark spin-flavor symmetry [6] (on the  $c$  and  $b$  quarks). Consequently, the leading corrections are suppressed by factors of the small quantity  $(m_s/m_c - m_s/m_b) \simeq 0.1$  or  $(m_s/1 \text{ GeV}) \cdot [\alpha_s(m_c)/\pi - \alpha_s(m_b)/\pi] \simeq 0.01$ , and a determination of  $|V_{ub}|$  with a theoretical uncertainty of about 10% is possible.

Semileptonic  $D \rightarrow K^* \bar{\ell} \nu_\ell$  decay ( $\ell = e, \mu$ ) has been studied extensively and the form-factors which characterize the hadronic  $D \rightarrow K^*$  matrix element of the weak current have been determined (with some assumptions concerning their shape) from the data. In this paper we denote the form-factors relevant for semileptonic transitions between a pseudoscalar meson containing a heavy quark,  $H$ , and a member of the lowest lying multiplet of vector mesons,  $V$ , by  $g^{(H \rightarrow V)}$ ,  $f^{(H \rightarrow V)}$  and  $a_\pm^{(H \rightarrow V)}$ , where

$$\langle V(p', \epsilon) | \bar{q} \gamma_\mu Q | H(p) \rangle = i g^{(H \rightarrow V)} \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} (p + p')^\lambda (p - p')^\sigma, \quad (1a)$$

$$\begin{aligned} \langle V(p', \epsilon) | \bar{q} \gamma_\mu \gamma_5 Q | H(p) \rangle &= f^{(H \rightarrow V)} \epsilon_\mu^* + a_+^{(H \rightarrow V)} (\epsilon^* \cdot p) (p + p')_\mu \\ &+ a_-^{(H \rightarrow V)} (\epsilon^* \cdot p) (p - p')_\mu, \end{aligned} \quad (1b)$$

and  $\varepsilon^{0123} = -\varepsilon_{0123} = 1$ . The sign of  $g$  depends on this convention for the Levi-Civita tensor. We view the form-factors  $g$ ,  $f$  and  $a_\pm$  as functions of the dimensionless variable  $y = v \cdot v'$ , where  $p = m_H v$ ,  $p' = m_V v'$ , and  $q^2 = (p - p')^2 = m_H^2 + m_V^2 - 2m_H m_V y$ . (Note that even though we are using the variable  $v \cdot v'$ , we are not treating the quarks in  $V$  as heavy.) The experimental values for the form-factors for  $D \rightarrow K^* \bar{\ell} \nu_\ell$  are [1]

$$f^{(D \rightarrow K^*)}(y) = \frac{1.8 \text{ GeV}}{1 + 0.63(y - 1)}, \quad (2a)$$

$$a_+^{(D \rightarrow K^*)}(y) = -\frac{0.17 \text{ GeV}^{-1}}{1 + 0.63(y - 1)}, \quad (2b)$$

$$g^{(D \rightarrow K^*)}(y) = -\frac{0.51 \text{ GeV}^{-1}}{1 + 0.96(y - 1)}. \quad (2c)$$

The form factor  $a_-$  is not measured because its contribution to the  $D \rightarrow K^* \bar{\ell} \nu_\ell$  decay amplitude is proportional to the lepton mass. The minimal value of  $y$  is unity (corresponding to the zero recoil point where the  $K^*$  is at rest in the  $D$  rest-frame) and the maximum value

of  $y$  is  $(m_D^2 + m_{K^*}^2)/(2m_D m_{K^*}) \simeq 1.3$  (corresponding to maximal  $K^*$  recoil in the  $D$  rest-frame). Note that over the whole kinematic range  $1 < y < 1.3$   $f$  changes by less than 20%. Therefore, in the following analysis of  $B$  decays we can extrapolate  $f$  with a small uncertainty to a somewhat larger region, which in what follows we take to be  $1 < y < 1.5$ . The full kinematic region for  $B \rightarrow \rho \ell \bar{\nu}_\ell$  is  $1 < y < 3.5$ .

## II. SEMILEPTONIC $B \rightarrow \rho \ell \bar{\nu}_\ell$ DECAY

The differential decay rate for semileptonic  $B$  decay (neglecting the lepton mass), not summed over the lepton type  $\ell$ , is

$$\frac{d\Gamma(B \rightarrow \rho \ell \bar{\nu}_\ell)}{dy} = \frac{G_F^2 |V_{ub}|^2}{48 \pi^3} m_B^3 r^2 S(y), \quad (3)$$

where  $r = m_\rho/m_B$  and  $S(y)$  is the function

$$\begin{aligned} S(y) &= \sqrt{y^2 - 1} \left[ \left| f^{(B \rightarrow \rho)}(y) \right|^2 (2 + y^2 - 6yr + 3r^2) \right. \\ &\quad + 4 \operatorname{Re} \left[ a_+^{(B \rightarrow \rho)}(y) f^{(B \rightarrow \rho)}(y) \right] m_B^2 r (y - r) (y^2 - 1) \\ &\quad + 4 \left| a_+^{(B \rightarrow \rho)}(y) \right|^2 m_B^4 r^2 (y^2 - 1)^2 + 8 \left| g^{(B \rightarrow \rho)}(y) \right|^2 m_B^4 r^2 (1 + r^2 - 2yr) (y^2 - 1) \left. \right] \\ &= \left| f^{(B \rightarrow \rho)}(y) \right|^2 [1 + \delta^{(B \rightarrow \rho)}(y)] \sqrt{y^2 - 1} (2 + y^2 - 6yr + 3r^2). \end{aligned} \quad (4)$$

The function  $\delta^{(B \rightarrow \rho)}$  depends on the ratios of form-factors  $a_+^{(B \rightarrow \rho)}/f^{(B \rightarrow \rho)}$  and  $g^{(B \rightarrow \rho)}/f^{(B \rightarrow \rho)}$ .

We can estimate  $S(y)$  using combinations of heavy quark symmetry and  $SU(3)$  flavor symmetry. Heavy quark symmetry implies the relations [7]

$$f^{(B \rightarrow K^*)}(y) = \left( \frac{m_B}{m_D} \right)^{1/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} f^{(D \rightarrow K^*)}(y), \quad (5a)$$

$$a_+^{(B \rightarrow K^*)}(y) = \frac{1}{2} \left( \frac{m_D}{m_B} \right)^{1/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} \left[ a_+^{(D \rightarrow K^*)}(y) \left( 1 + \frac{m_c}{m_b} \right) - a_-^{(D \rightarrow K^*)}(y) \left( 1 - \frac{m_c}{m_b} \right) \right], \quad (5b)$$

$$g^{(B \rightarrow K^*)}(y) = \left( \frac{m_D}{m_B} \right)^{1/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} g^{(D \rightarrow K^*)}(y). \quad (5c)$$

$SU(3)$  symmetry implies that the  $\bar{B}^0 \rightarrow \rho^+$  form-factors are equal to the  $B \rightarrow K^*$  form-factors and the  $B^- \rightarrow \rho^0$  form-factors are equal to  $1/\sqrt{2}$  times the  $B \rightarrow K^*$  form-factors.

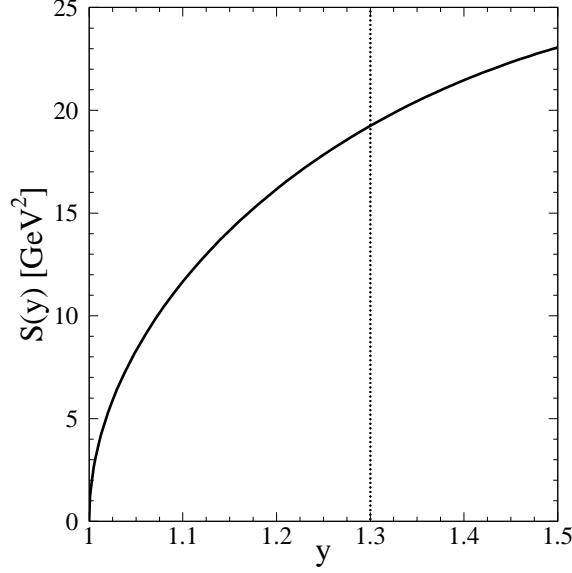


FIG. 1. The function  $S(y)$  defined in eq. (4) as a function of the kinematic variable  $y = v \cdot v'$ . The dotted vertical line corresponds to the kinematic limit for  $D \rightarrow K^* \bar{\ell} \nu_\ell$ .

In the limit where the heavy quark  $Q$  has large mass, the matrix elements in eqs. (1) depend on  $m_Q$  only through a factor of  $\sqrt{m_H}$  associated with the normalization of the heavy meson states. Consequently, for large  $m_c$ ,  $(a_+^{(D \rightarrow K^*)} + a_-^{(D \rightarrow K^*)})/(a_+^{(D \rightarrow K^*)} - a_-^{(D \rightarrow K^*)})$  is of order  $\Lambda_{\text{QCD}}/m_c$ , so we can set  $a_-^{(D \rightarrow K^*)} = -a_+^{(D \rightarrow K^*)}$  in eq. (5b), yielding

$$a_+^{(B \rightarrow K^*)}(y) = \left(\frac{m_D}{m_B}\right)^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} a_+^{(D \rightarrow K^*)}(y). \quad (6)$$

Using eqs. (5a), (5c), (6) and  $SU(3)$  to get the  $\bar{B}^0 \rightarrow \rho^+ \ell \bar{\nu}_\ell$  form-factors (in the region  $1 < y < 1.5$ ) from those for  $D \rightarrow K^* \bar{\ell} \nu_\ell$ , given in eq. (2), gives  $S(y)$  plotted in Fig. 1. We use  $\alpha_s(m_b) = 0.22$  and  $\alpha_s(m_c) = 0.39$ . In Fig. 2 we plot  $\delta^{(B \rightarrow \rho)}(y)$  and  $\delta^{(B \rightarrow K^*)}(y)$  as a function of  $y$ . The latter function (which will be used later in this paper) is denoted by the dashed curve. Perhaps the largest uncertainty in our analysis for  $\delta$  comes from setting  $a_-^{(D \rightarrow K^*)} = -a_+^{(D \rightarrow K^*)}$ . If  $a_-^{(D \rightarrow K^*)} = -\lambda a_+^{(D \rightarrow K^*)}$ , then eq. (6) gets multiplied on its right hand side by the factor  $(1 + m_D/m_B)/2 + \lambda(1 - m_D/m_B)/2$ . In Fig. 3 we plot  $\delta^{(B \rightarrow \rho)}$  and  $\delta^{(B \rightarrow K^*)}$  for  $\lambda = 0$  and 2.

Note that  $\delta$  is fairly small, indicating that  $a_+^{(B \rightarrow \rho)}$  and  $g^{(B \rightarrow \rho)}$  make a small contribution to  $S(y)$  (in the region  $1 < y < 1.5$ ), so even significant corrections to eq. (6) will not have

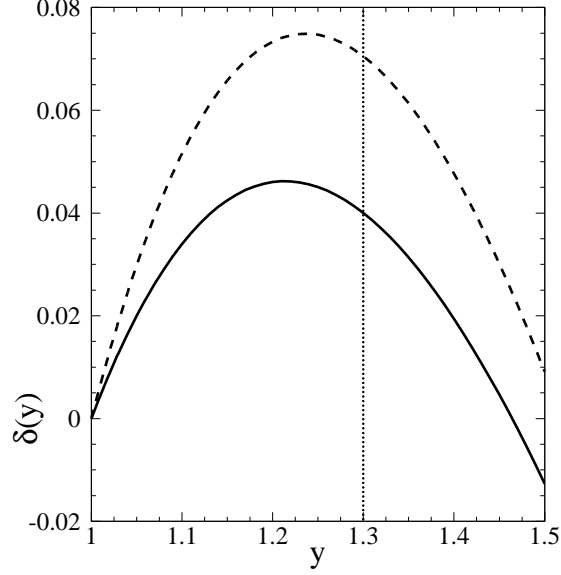


FIG. 2. The function  $\delta(y)$  as a function of the kinematic variable  $y = v \cdot v'$ . The solid curve is  $\delta^{(B \rightarrow \rho)}(y)$ , the dashed curve is  $\delta^{(B \rightarrow K^*)}(y)$ .

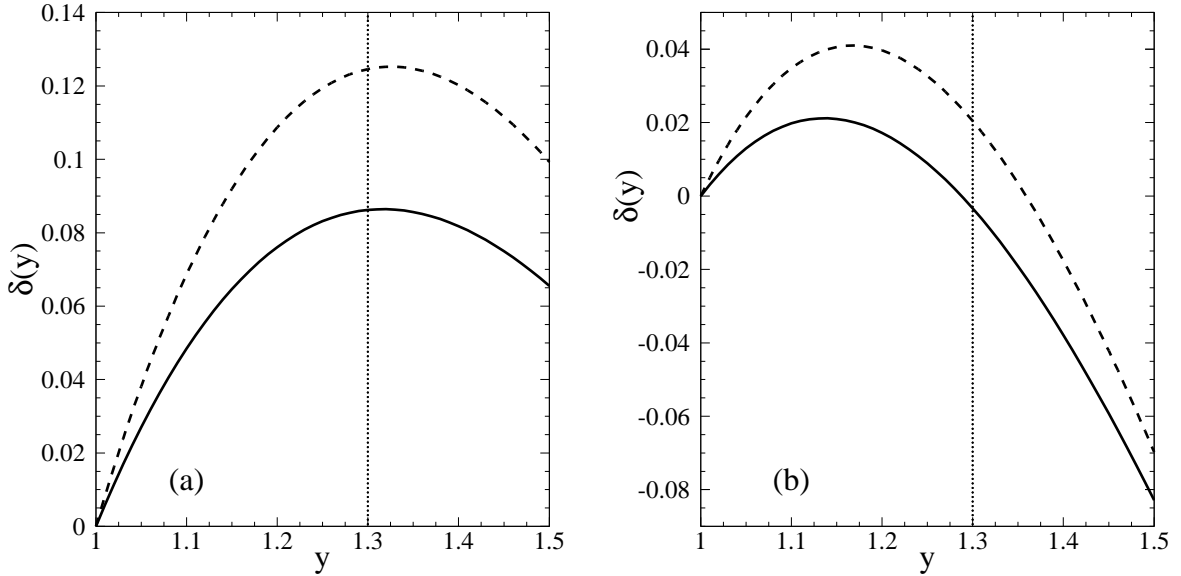


FIG. 3. The function  $\delta(y)$  as a function of the kinematic variable  $y = v \cdot v'$ . Fig. 3a corresponds to  $\lambda = 0$ , Fig. 3b to  $\lambda = 2$ . The solid curves are  $\delta^{(B \rightarrow \rho)}(y)$ , the dashed curves are  $\delta^{(B \rightarrow K^*)}(y)$ .

a large impact on  $S(y)$ . We can use our prediction for  $S(y)$  to determine  $|V_{ub}|$  from the  $B \rightarrow \rho \ell \bar{\nu}_\ell$  semileptonic decay rate in the region  $1 < y < 1.5$ . Our predicted  $S(y)$ , in Fig. 1, gives a branching ratio of  $5.2 |V_{ub}|^2$  for  $\bar{B}^0 \rightarrow \rho^+ \ell \bar{\nu}_\ell$  in the region  $1 < y < 1.5$  (corresponding to  $16 \text{ GeV}^2 < q^2 < q_{\text{max}}^2 = 20 \text{ GeV}^2$ , which implies  $E_\ell > 1.6 \text{ GeV}$  in the  $B$  rest-frame). While such a model-independent determination of  $|V_{ub}|$  may eventually be superior to a determination from a comparison of the endpoint of the electron spectrum with phenomenological models [8,9], there will be a sizable theoretical uncertainty associated with  $|V_{ub}|$  determined in this way from order  $m_s$   $SU(3)$  violation and order  $1/m_{c,b}$  corrections to relations (5) and (6). What is needed to get a value for  $|V_{ub}|$  with smaller theoretical uncertainties is an improved method for determining  $|f^{(B \rightarrow \rho)}|^2 (1 + \delta^{(B \rightarrow \rho)})$ .

Our method for determining a precise value for  $|V_{ub}|$  is based on the observation that the ‘‘Grinstein-type double ratio’’ [10]  $(f^{(B \rightarrow \rho)}/f^{(B \rightarrow K^*)})/(f^{(D \rightarrow \rho)}/f^{(D \rightarrow K^*)})$  is equal to unity in three separate limits of QCD (isospin violation is neglected here): (i) the limit of  $SU(3)$  flavor symmetry,  $m_s \rightarrow 0$ , where the strange quark mass is treated as small compared with a typical hadronic scale; (ii) the limit of  $SU(4)$  heavy quark spin-flavor symmetry,  $m_{b,c} \rightarrow \infty$ , where the bottom and charm quark masses are treated as large compared with a typical hadronic scale; (iii) the limit  $m_c = m_b$ , where the bottom and the charm quarks are related by an  $SU(2)$  flavor symmetry. Consequently,

$$f^{(B \rightarrow \rho)} = f^{(B \rightarrow K^*)} \frac{f^{(D \rightarrow \rho)}}{f^{(D \rightarrow K^*)}} \left[ 1 + \mathcal{O} \left( \frac{m_s}{m_c} - \frac{m_s}{m_b}, \frac{m_s}{1 \text{ GeV}} \frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi} \right) \right]. \quad (7)$$

We propose to extract a precise value for  $|f^{(B \rightarrow \rho)}|^2 (1 + \delta^{(B \rightarrow \rho)})$  using

$$\left| f^{(B \rightarrow \rho)} \right|^2 (1 + \delta^{(B \rightarrow \rho)}) = \left| f^{(B \rightarrow K^*)} \right|^2 (1 + \delta^{(B \rightarrow K^*)}) \left| \frac{f^{(D \rightarrow \rho)}}{f^{(D \rightarrow K^*)}} \right|^2. \quad (8)$$

Multiplying by the ratio of  $D$  decay form-factors above cancels out  $SU(3)$  violation not suppressed by factors of the heavy quark mass in the most important part of the  $B \rightarrow \rho \ell \bar{\nu}_\ell$  differential decay rate, *i.e.*, the factor of  $|f^{(B \rightarrow \rho)}|^2$ , leaving an uncertainty from  $SU(3)$  violation only in  $\delta$ . Since as we have argued,  $|\delta|$  is likely to be less than 0.15, the effects of  $SU(3)$  violation in it can safely be neglected. The plots in Figs. 2 and 3 show the kinematic

sources of  $SU(3)$  violation in  $\delta$  arising from the fact that the  $\rho$  and  $K^*$  masses are not equal. There are also contributions from  $SU(3)$  violation in the ratios of the form-factors  $a_+/f$  and  $g/f$ .

In principle, the form factor  $f^{(D \rightarrow \rho)}$  can be obtained from experimental information on the Cabibbo suppressed decay  $D \rightarrow \rho \bar{\ell} \nu_\ell$ . However, at the present time, the small branching ratio [1]  $\text{Br}(D^+ \rightarrow \rho^0 \bar{\mu} \nu_\mu) = (2.0^{+1.5}_{-1.3}) \times 10^{-3}$  has made extraction of the form factor  $f^{(D \rightarrow \rho)}$  too difficult. It may be possible in future fixed target experiments or at a tau-charm factory to determine  $f^{(D \rightarrow \rho)}$ . Assuming this can be done, the factor  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  is the remaining ingredient needed for a determination of  $|f^{(B \rightarrow \rho)}|^2 (1 + \delta^{(B \rightarrow \rho)})$  via eq. (8).

### III. RARE $B$ DECAYS

One avenue to find the factor  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  uses the exclusive rare decays  $B \rightarrow K^* \ell \bar{\ell}$  or  $B \rightarrow K^* \nu \bar{\nu}$ , which may eventually be studied at hadron colliders, or at  $B$  factories. The effective Hamiltonian for these decays is [11–14]

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum C_i(\mu) O_i(\mu), \quad (9)$$

where  $\mu$  is the subtraction point (hereafter we set  $\mu = m_b$  and do not explicitly display the subtraction point dependence), and the operators  $O_i$  are:

$$O_1 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha}) (\bar{c}_{L\beta} \gamma^\mu c_{L\beta}), \quad (10a)$$

$$O_2 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\beta}) (\bar{c}_{L\beta} \gamma^\mu c_{L\alpha}), \quad (10b)$$

$$O_3 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha}) \left[ (\bar{u}_{L\beta} \gamma^\mu u_{L\beta}) + \dots + (\bar{b}_{L\beta} \gamma^\mu b_{L\beta}) \right], \quad (10c)$$

$$O_4 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\beta}) \left[ (\bar{u}_{L\beta} \gamma^\mu u_{L\alpha}) + \dots + (\bar{b}_{L\beta} \gamma^\mu b_{L\alpha}) \right], \quad (10d)$$

$$O_5 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha}) \left[ (\bar{u}_{R\beta} \gamma^\mu u_{R\beta}) + \dots + (\bar{b}_{R\beta} \gamma^\mu b_{R\beta}) \right], \quad (10e)$$

$$O_6 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\beta}) \left[ (\bar{u}_{R\beta} \gamma^\mu u_{R\alpha}) + \dots + (\bar{b}_{R\beta} \gamma^\mu b_{R\alpha}) \right], \quad (10f)$$

$$O_7 = (e/16\pi^2) m_b (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, \quad (10g)$$

$$O_8 = (g/16\pi^2) m_b (\bar{s}_L \sigma_{\mu\nu} b_R) G^{\mu\nu}, \quad (10h)$$



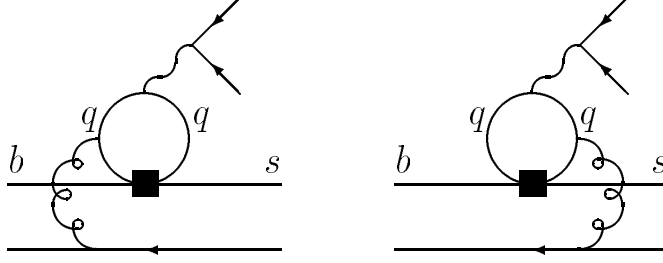


FIG. 4. Feynman diagrams whose contribution to exclusive rates is neither included in the form-factors, nor in the effective Wilson coefficient  $\tilde{C}_9$ . The black square represents one of the four-quark operators  $O_1 - O_6$ .

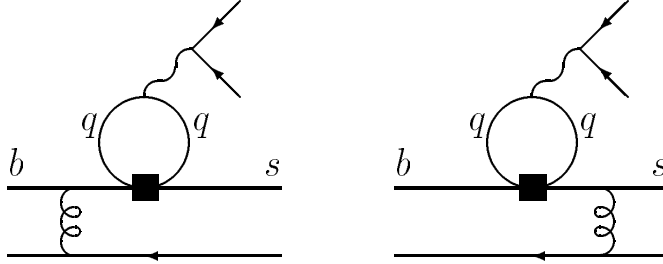


FIG. 5. Feynman diagrams whose contribution to exclusive rates is part of the nonperturbative matrix element of  $\tilde{C}_9 O_9$ .

$$O_9 = (e^2/16\pi^2) (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell), \quad (10i)$$

$$O_{10} = (e^2/16\pi^2) (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell), \quad (10j)$$

$$O_{11} = (e^2/16\pi^2 \sin^2 \theta_W) (\bar{s}_L \gamma_\mu b_L) [\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu]. \quad (10k)$$

For  $B \rightarrow K^* \ell \bar{\ell}$  we need the matrix elements of  $O_1 - O_6$  and  $O_8$  at order  $e^2$  and to all orders in the strong interactions, and the matrix elements of  $O_7$ ,  $O_9$  and  $O_{10}$  to all orders in the strong interactions. Among the contributions to the  $B \rightarrow K^* \ell \bar{\ell}$  matrix element of  $O_1 - O_6$  are the Feynman diagrams in Fig. 4, where a soft gluon (with momentum of order  $k \ll \sqrt{q^2}$ ) connects to the  $q\bar{q}$  loop. We are interested in the kinematic region  $1 < y < 1.5$  which corresponds to a  $\ell\bar{\ell}$  pair with large invariant mass squared  $q^2$  between  $14.5 \text{ GeV}^2$  and  $19 \text{ GeV}^2$ . In this kinematic region we have found by explicit computation that the contribution of the Feynman diagrams in Fig. 4 are suppressed by at least a factor of  $k/\sqrt{q^2}$  compared, for example, to the contributions of the diagrams in Fig. 5. In the region of large

$q^2$  (compared with the QCD scale and the mass of the quark  $q$ ) the  $q\bar{q}$  pair must “quickly” convert into the (color singlet)  $\ell\bar{\ell}$  pair and hence the coupling of soft, long wavelength gluons to the  $q\bar{q}$  pair is suppressed at all orders in QCD perturbation theory. Similar remarks hold for the matrix elements of  $O_8$ . This “factorization conjecture” implies that for  $B \rightarrow K^* \ell \bar{\ell}$  at large  $q^2$  we can take the matrix elements of  $O_1 - O_6$  and  $O_8$  into account by adjusting the coefficients of  $O_7$  and  $O_9$  by a calculable short distance correction. In the next-to-leading logarithmic approximation  $C_9$  is replaced by an effective  $\tilde{C}_9(y)$  coupling [13]

$$\begin{aligned} \tilde{C}_9(y) = & C_9 + h(z, y) (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2} h(0, y) (C_3 + 3C_4) \\ & - \frac{1}{2} h(1, y) (4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6). \end{aligned} \quad (11)$$

Here

$$\begin{aligned} h(z, y) = & -\frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x \\ & - \frac{2}{9} (2 + x) \sqrt{|1 - x|} \begin{cases} \left( \ln \left| \frac{\sqrt{|1-x|}+1}{\sqrt{|1-x|-1}} \right| - i\pi \right); & \text{for } x \equiv 4m_c^2/q^2 < 1 \\ 2 \arctan(1/\sqrt{x-1}); & \text{for } x \equiv 4m_c^2/q^2 > 1, \end{cases} \end{aligned} \quad (12)$$

with  $h(0, y) = 8/27 - (4/9) [\ln(q^2/m_b^2) - i\pi]$ , and  $z = m_c/m_b$ ,  $r = m_{K^*}/m_B$ . On the right hand side of eq. (12)  $q^2 = m_B^2 + m_{K^*}^2 - 2m_B m_{K^*} y$  should be understood. Fig. 5 is now part of the nonperturbative matrix element of  $\tilde{C}_9 O_9$ . Note that eq. (11) differs from Ref. [13], since the one gluon correction to the matrix element of  $O_9$  is viewed as a contribution to the form-factors in our case.

Using  $m_t = 175 \text{ GeV}$ ,  $m_b = 4.8 \text{ GeV}$ ,  $m_c = 1.4 \text{ GeV}$ ,  $\alpha_s(m_W) = 0.12$ ,  $\alpha_s(m_b) = 0.22$  and  $\sin^2 \theta_W = 0.23$ , the numerical values of the Wilson coefficients in the leading logarithmic approximation are  $C_1 = -0.26$ ,  $C_2 = 1.11$ ,  $C_3 = 0.01$ ,  $C_4 = -0.03$ ,  $C_5 = 0.008$ ,  $C_6 = -0.03$ ,  $C_7 = -0.32$ . The operator  $O_8$  does not contribute at the order we are working.  $C_9$ ,  $C_{10}$  and  $C_{11}$  depend more sensitively on  $m_t$  (quadratically for  $m_t \gg m_W$ ). In Table I we give their values for  $m_t = 165 \text{ GeV}$ ,  $m_t = 175 \text{ GeV}$  and  $m_t = 185 \text{ GeV}$ .

In eq. (11) the second term on the right hand side, proportional to  $h(z, y)$  comes from charm quark loops. Since  $q^2$  is close to  $4m_c^2$ , one is not in a kinematic region where the

	$m_t = 165 \text{ GeV}$	$m_t = 175 \text{ GeV}$	$m_t = 185 \text{ GeV}$
$C_9$	4.17	4.26	4.34
$C_{10}$	-4.21	-4.62	-5.04
$C_{11}$	1.40	1.48	1.57

TABLE I. Coefficients of the  $O_9 - O_{11}$  operators at the scale  $m_b$  for different values of the top quark mass.  $C_{10}$  is calculated in the leading logarithmic approximation, while  $C_9$  and  $C_{11}$  are calculated to next-to-leading order accuracy.\*

perturbative QCD calculation of the  $c\bar{c}$  loop (or factorization) can be trusted. Threshold effects, which spoil local duality, may be important. (In the kinematic region near  $q^2 = 0$  the charm quarks in the loop are far off-shell and eq. (12) should be valid. However, in this region we cannot justify using eq. (11) for the light quark loops.) Later we examine the sensitivity of the  $B \rightarrow K^* \ell \bar{\ell}$  rate in the kinematic region of interest to  $c\bar{c}$  threshold effects. For slightly lower values of  $q^2$  (or equivalently for larger values of  $y$ ) than we consider, such effects are very important. The rates for  $B \rightarrow K^* J/\psi \rightarrow K^* \ell \bar{\ell}$  and for  $B \rightarrow K^* \psi' \rightarrow K^* \ell \bar{\ell}$  are much greater than what eq. (11) would imply. The latter process occurs with the  $\psi'$  on mass-shell at  $y = 1.6$ .

The hadronic matrix element of  $O_7$  is expressed in terms of new hadronic form-factors,  $g_{\pm}$  and  $h$ , defined by

$$\begin{aligned} \langle V(p', \epsilon) | \bar{q} \sigma_{\mu\nu} Q | H(p) \rangle &= g_+^{(H \rightarrow V)} \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (p + p')^\sigma + g_-^{(H \rightarrow V)} \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (p - p')^\sigma \\ &+ h^{(H \rightarrow V)} \varepsilon_{\mu\nu\lambda\sigma} (p + p')^\lambda (p - p')^\sigma (\epsilon^* \cdot p), \end{aligned} \quad (13a)$$

$$\begin{aligned} \langle V(p', \epsilon) | \bar{q} \sigma_{\mu\nu} \gamma_5 Q | H(p) \rangle &= i g_+^{(H \rightarrow V)} [\epsilon_\nu^* (p + p')_\mu - \epsilon_\mu^* (p + p')_\nu] \\ &+ i g_-^{(H \rightarrow V)} [\epsilon_\nu^* (p - p')_\mu - \epsilon_\mu^* (p - p')_\nu] \\ &+ i h^{(H \rightarrow V)} [(p + p')_\nu (p - p')_\mu - (p + p')_\mu (p - p')_\nu] (\epsilon^* \cdot p). \end{aligned} \quad (13b)$$

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\*For  $C_9$  in the next-to-leading logarithmic approximation terms of order  $\alpha_s$  are subdominant, since the leading contribution to  $C_9$  is order  $\ln(m_W^2/m_b^2) \sim 1/\alpha_s$ .

The second relation is obtained from the first one using  $\sigma^{\mu\nu} = \frac{i}{2} \varepsilon^{\mu\nu\lambda\sigma} \sigma_{\lambda\sigma} \gamma_5$ . The differential decay rate for  $B \rightarrow K^* \ell \bar{\ell}$  (not summed over the lepton type  $\ell$ ) is

$$\frac{d\Gamma(B \rightarrow K^* \ell \bar{\ell})}{dy} = \frac{G_F^2 |V_{ts}^* V_{tb}|^2}{24 \pi^3} \left( \frac{\alpha}{4\pi} \right)^2 m_B^3 r^2 \left[ |\tilde{C}_9(y)|^2 S'(y) + |C_{10}|^2 S(y) \right], \quad (14)$$

where  $S(y)$  is given by the expression in eq. (4), with the form-factors replaced by those appropriate for  $B \rightarrow K^*$ , and  $r = m_{K^*}/m_B$ .  $S'(y)$  is obtained from  $S(y)$  via the replacements

$$f^{(B \rightarrow K^*)} \rightarrow f^{(B \rightarrow K^*)} + \left[ g_+^{(B \rightarrow K^*)} (m_B^2 - m_{K^*}^2) + g_-^{(B \rightarrow K^*)} m_B^2 (1 + r^2 - 2yr) \right] A(y), \quad (15a)$$

$$a_+^{(B \rightarrow K^*)} \rightarrow a_+^{(B \rightarrow K^*)} + \left[ h^{(B \rightarrow K^*)} m_B^2 (1 + r^2 - 2yr) - g_+^{(B \rightarrow K^*)} \right] A(y), \quad (15b)$$

$$g^{(B \rightarrow K^*)} \rightarrow g^{(B \rightarrow K^*)} - g_+^{(B \rightarrow K^*)} A(y), \quad (15c)$$

where  $A(y) = 2m_b C_7 / [m_B^2 (1 + r^2 - 2yr) \tilde{C}_9(y)]$ . Since  $C_7$  is small compared to  $\tilde{C}_9$ , it is convenient to rewrite the differential decay rate as

$$\begin{aligned} \frac{d\Gamma(B \rightarrow K^* \ell \bar{\ell})}{dy} &= \frac{G_F^2 |V_{ts}^* V_{tb}|^2}{24 \pi^3} \left( \frac{\alpha}{4\pi} \right)^2 m_B^3 r^2 \left[ |\tilde{C}_9(y)|^2 + |C_{10}|^2 \right] \\ &\times \left| f^{(B \rightarrow K^*)}(y) \right|^2 \left[ 1 + \delta^{(B \rightarrow K^*)}(y) \right] \sqrt{y^2 - 1} (2 + y^2 - 6yr + 3r^2) [1 + \Delta(y)], \end{aligned} \quad (16)$$

where  $\Delta$  contains the dependence of the differential decay rate on  $C_7$ .

Unitarity of the CKM matrix implies that  $|V_{ts}^* V_{tb}| \simeq |V_{cs}^* V_{cb}|$  (with no more than 3% uncertainty), so that once  $\Delta(y)$  is known, a value of  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  can be determined from experimental data on  $B \rightarrow K^* \ell \bar{\ell}$ . To find  $\Delta(y)$  we use the relations between the tensor and (axial-)vector form-factors derived for large  $m_b$  in Ref. [7]<sup>†</sup>

$$g_+^{(B \rightarrow K^*)} + g_-^{(B \rightarrow K^*)} = \frac{f^{(B \rightarrow K^*)} + 2 g^{(B \rightarrow K^*)} m_B m_{K^*} y}{m_B}, \quad (17a)$$

$$g_+^{(B \rightarrow K^*)} - g_-^{(B \rightarrow K^*)} = -2 m_B g^{(B \rightarrow K^*)}, \quad (17b)$$

$$h^{(B \rightarrow K^*)} = \frac{a_+^{(B \rightarrow K^*)} - a_-^{(B \rightarrow K^*)} - 2 g^{(B \rightarrow K^*)}}{2 m_B}. \quad (17c)$$

Recent lattice QCD simulations indicate that these relations hold within 20% accuracy at the scale of the  $B$  mass [15]. In the limit where  $m_b$  is treated as heavy,  $a_+^{(B \rightarrow K^*)} + a_-^{(B \rightarrow K^*)}$  is small compared with  $a_+^{(B \rightarrow K^*)} - a_-^{(B \rightarrow K^*)}$ , so eq. (17c) can be simplified to

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<sup>†</sup>We correct some obvious factor-of-two errors in [7].

$$h^{(B \rightarrow K^*)} = \frac{a_+^{(B \rightarrow K^*)} - g^{(B \rightarrow K^*)}}{m_B}. \quad (18)$$

Note that a similar simplification for  $g_+^{(B \rightarrow K^*)} + g_-^{(B \rightarrow K^*)}$  is not useful, because in eq. (15a)  $g_+^{(B \rightarrow K^*)} + g_-^{(B \rightarrow K^*)}$  is enhanced by  $m_B$  compared to  $g_+^{(B \rightarrow K^*)} - g_-^{(B \rightarrow K^*)}$ .

Using eqs. (14), (15), (16), (17a), (17b) and (18),  $\Delta(y)$  is expressed in terms of  $C_7$ ,  $\tilde{C}_9$ ,  $C_{10}$ ,  $g^{(B \rightarrow K^*)}/f^{(B \rightarrow K^*)}$  and  $a_+^{(B \rightarrow K^*)}/f^{(B \rightarrow K^*)}$ . Using eqs. (5) and (6) to relate ratios of  $B \rightarrow K^*$  form-factors to ratios of  $D \rightarrow K^*$  form-factors, we find that in the kinematic region  $1 < y < 1.5$ ,  $\Delta(y)$  changes almost linearly from  $\Delta(1) \simeq -0.14$  to  $\Delta(1.5) \simeq -0.18$ . The value of  $\Delta$  at zero recoil (using  $m_b \simeq m_B$ ) does not depend on the ratios of form-factors [16]

$$\Delta(1) = \frac{1}{|\tilde{C}_9(1)|^2 + |C_{10}|^2} \left[ \frac{4 \operatorname{Re} [C_7^* \tilde{C}_9(1)]}{1 - r} + \frac{4 |C_7|^2}{(1 - r)^2} \right]. \quad (19)$$

Even though there are  $1/m_c$  corrections to eqs. (5) and (6), they do not affect  $\Delta(1)$ . Furthermore,  $\Delta$  is small compared with unity and has a modest  $y$ -dependence. Consequently,  $1/m_c$  corrections to the  $y$ -dependence of  $\Delta$ , and  $1/m_b$  corrections to  $\Delta(1)$  can only have a very small impact on a value of  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  extracted from the  $B \rightarrow K^* \ell \bar{\ell}$  differential decay rate using eq. (16).

Using the measured values of the  $D \rightarrow K^* \bar{\ell} \nu_\ell$  form factors and the heavy quark symmetry relations in eqs. (5) and (6) to get  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$ , together with  $|V_{cb}| = 0.04$ ,  $\tau_B = 1.5 \text{ ps}$  and  $\alpha(m_W) = 1/129$ , we find that eq. (16) gives a branching ratio of  $2.9 \times 10^{-7}$  for  $B \rightarrow K^* \ell \bar{\ell}$  in the kinematic region  $1 < y < 1.5$ .

The largest theoretical uncertainties in using  $B \rightarrow K^* \ell \bar{\ell}$  for extracting  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  come from order  $\alpha_s$  corrections to the coefficients of the operators  $O_9$  and  $O_{10}$  and our treatment of the  $B \rightarrow K^* \ell \bar{\ell}$  matrix element of the four-quark operators. It is  $h(z, y)$  that takes into account the  $c\bar{c}$  loop contributions to the matrix elements of the four-quark operators.

A comparison with a phenomenological resonance saturation model [17] gives an indication of the uncertainties in the prediction for  $B \rightarrow K^* \ell \bar{\ell}$  that arise from the fact that the kinematic region we focus on is not far from  $D\bar{D}$  threshold. In this regard we note that

	$M_{\psi^{(n)}} [\text{GeV}]$	$\Gamma_{\psi^{(n)}} [\text{GeV}]$	$\text{Br}(\psi^{(n)} \rightarrow \ell \bar{\ell})$
$\psi^{(1)} = J/\psi$	3.097	$8.8 \cdot 10^{-5}$	$6.0 \cdot 10^{-2}$
$\psi^{(2)}$	3.686	$2.8 \cdot 10^{-4}$	$8.4 \cdot 10^{-3}$
$\psi^{(3)}$	3.77	$2.4 \cdot 10^{-2}$	$1.1 \cdot 10^{-5}$
$\psi^{(4)}$	4.04	$5.2 \cdot 10^{-2}$	$1.4 \cdot 10^{-5}$
$\psi^{(5)}$	4.16	$7.8 \cdot 10^{-2}$	$1.0 \cdot 10^{-5}$
$\psi^{(6)}$	4.42	$4.3 \cdot 10^{-2}$	$1.1 \cdot 10^{-5}$

TABLE II. Mass, width and leptonic branching ratio of the  $1^{--} c\bar{c}$  resonances [1].

using factorization to estimate the  $B \rightarrow K^* \psi^{(n)} \rightarrow K^* \ell \bar{\ell}$  matrix elements of the four-quark operators ( $\psi^{(n)}$  is the  $n$ 'th  $1^{--} c\bar{c}$  resonance) we find that in a resonance saturation model  $h(z, y)$  in the second term of eq. (11) gets replaced by<sup>‡</sup>

$$h(z, y) \rightarrow -\kappa \frac{3\pi}{\alpha^2} \sum_n \frac{\Gamma_{\psi^{(n)}} \text{Br}(\psi^{(n)} \rightarrow \ell \bar{\ell})}{(q^2 - M_{\psi^{(n)}}^2)/M_{\psi^{(n)}} + i\Gamma_{\psi^{(n)}}}, \quad (20)$$

where  $\Gamma_{\psi^{(n)}}$  and  $M_{\psi^{(n)}}$  are the width and mass of the  $n$ 'th  $1^{--} c\bar{c}$  resonance. Experimental values for these quantities and the branching ratios to  $\ell\bar{\ell}$  are given in Table II. In eq. (20)  $\kappa = 2.3 e^{i\varphi_\kappa}$  is the factor that the  $B \rightarrow J/\psi K^*$  amplitude, calculated using naive factorization, must be multiplied by to get the measured  $B \rightarrow J/\psi K^*$  rate. Since the magnitude of  $\kappa$  is large, we do not assume that eq. (20) has the same phase (*i.e.*,  $\varphi_\kappa = 0$ ) as naive factorization would imply. Replacing  $h(z, y)$  in eq. (11) by the expression in eq. (20) results in an effective coefficient of  $O_9$  that we call  $\tilde{C}'_9$ . A measure of the deviation of this model for the  $c\bar{c}$  resonance region from the expression in eq. (11) is given by  $d(y)$  defined by

$$|\tilde{C}'_9(y)|^2 + |C_{10}|^2 = (|\tilde{C}_9(y)|^2 + |C_{10}|^2) [1 + d(y)]. \quad (21)$$

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<sup>‡</sup>For  $q^2$  not near the resonances, there are uncertainties associated with the  $q^2$  dependence. In eq. (20) factors of  $q^2$  not associated with the resonance propagator are set equal to the square of the resonance mass.

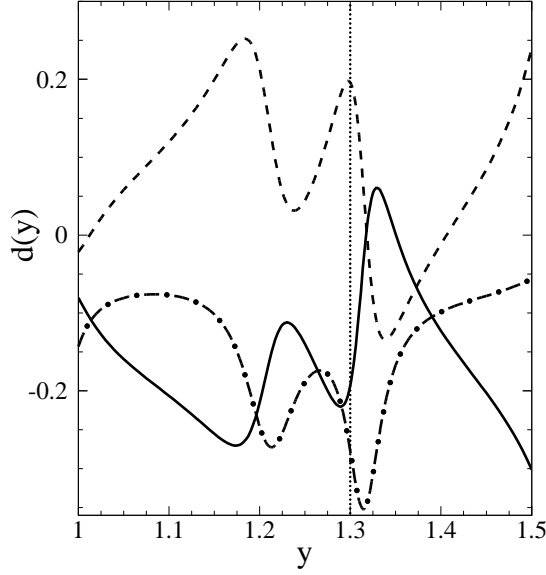


FIG. 6. The function  $d(y)$  defined in eq. (21) as a function of the kinematic variable  $y = v \cdot v'$ . The solid, dash-dotted and dashed curves correspond respectively to  $\varphi_\kappa = 0, \pi/2$  and  $\pi$ .

In Fig. 6 we plot  $d(y)$  for  $1 < y < 1.5$ . Note that part of  $h(z, y)$  is associated with  $c\bar{c}$  pairs at large virtuality, and so is reliably reproduced by QCD perturbation theory. In fact  $h(z, y)$  is scheme dependent, and so  $d(y)$  is only a very crude measure of the uncertainties that arise from being near the  $c\bar{c}$  threshold. The solid, dash-dotted and dashed curves in Fig. 6 correspond respectively to  $\varphi_\kappa = 0, \pi/2$  and  $\pi$ . This analysis suggests that the uncertainty associated with the charm threshold region has on average about a 20% effect on the  $B \rightarrow K^* \ell \bar{\ell}$  rate for  $1 < y < 1.5$ .

The uncertainties, involving the  $D\bar{D}$  threshold region and the order  $\alpha_s$  contributions to  $C_9$  and  $C_{10}$ , can be avoided if the decay  $B \rightarrow K^* \nu \bar{\nu}$  can be studied experimentally. While this will be difficult, the large missing energy carried by the neutrinos in the kinematic region we are interested in may help [18]. The differential decay rate for  $B \rightarrow K^* \nu \bar{\nu}$  (summed over the neutrino flavors) is

$$\begin{aligned}
\frac{d\Gamma(B \rightarrow K^* \nu \bar{\nu})}{dy} &= \frac{G_F^2 |V_{ts}^* V_{tb}|^2}{16 \pi^3} \left( \frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 m_B^3 r^2 |C_{11}|^2 S(y) \\
&= \frac{G_F^2 |V_{ts}^* V_{tb}|^2}{16 \pi^3} \left( \frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 m_B^3 r^2 |C_{11}|^2 \\
&\quad \times \left| f^{(B \rightarrow K^*)}(y) \right|^2 [1 + \delta^{(B \rightarrow K^*)}(y)] \sqrt{y^2 - 1} (2 + y^2 - 6yr + 3r^2). \quad (22)
\end{aligned}$$

The coefficient  $C_{11}$  depends on the top quark mass (see Table I). Once the top quark mass is known more accurately, the  $B \rightarrow K^* \nu \bar{\nu}$  differential decay rate provides a way to get  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  that, from a theoretical perspective, is very clean. Recall that the function  $\delta^{(B \rightarrow K^*)}$  is the analog of  $\delta^{(B \rightarrow \rho)}$  that occurred in  $B \rightarrow \rho \ell \bar{\nu}_\ell$  semileptonic decay, but it depends on ratios of  $B \rightarrow K^*$  form-factors that occur, instead of  $B \rightarrow \rho$  form-factors. It is plotted in Fig. 2 with the dashed curve using eqs. (5) and (6) to deduce the ratios of form-factors  $a_+^{(B \rightarrow K^*)}/f^{(B \rightarrow K^*)}$  and  $g^{(B \rightarrow K^*)}/f^{(B \rightarrow K^*)}$  from the  $D \rightarrow K^* \bar{\ell} \nu_\ell$  form-factors.  $\delta^{(B \rightarrow K^*)}(y)$  is fairly small, and so even though there is  $SU(3)$  violation in the relation between  $\delta^{(B \rightarrow K^*)}$  and  $\delta^{(B \rightarrow \rho)}$ , this does not introduce a large uncertainty in our prediction for  $|f^{(B \rightarrow \rho)}|^2 (1 + \delta^{(B \rightarrow \rho)})$  using eq. (8). Using eqs. (5) and (6) to get  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  from the measured values of the  $D \rightarrow K^* \bar{\ell} \nu_\ell$  form-factors, we find that eq. (22) implies a branching ratio of  $1.9 \times 10^{-6}$  for  $B \rightarrow K^* \nu \bar{\nu}$  in the kinematic region  $1 < y < 1.5$ .

The difference in the factor  $\sqrt{y^2 - 1} (2 + y^2 - 6yr + 3r^2)$  for  $r = m_\rho/m_B$  and  $r = m_{K^*}/m_B$  divided by their sum is less than 3% for  $1 < y < 1.5$ . Therefore, it is a good approximation to rewrite eq. (8), using eqs. (3), (4) and (22), as

$$\frac{d\Gamma(B \rightarrow \rho \ell \bar{\nu}_\ell)}{dy} = \frac{|V_{ub}|^2}{3|V_{ts}^* V_{tb}|^2} \left( \frac{2\pi \sin^2 \theta_W}{\alpha |C_{11}|} \right)^2 \frac{m_\rho^2}{m_{K^*}^2} \frac{d\Gamma(B \rightarrow K^* \nu \bar{\nu})}{dy} \left| \frac{f^{(D \rightarrow \rho)}(y)}{f^{(D \rightarrow K^*)}(y)} \right|^2. \quad (23)$$

If  $SU(3)$  violation in the  $y$ -dependence of the ratio of  $D$  decay form-factors in eq. (23) is small then we can also compare integrated  $B$  decay rates to get a precise value for  $|V_{ub}|$ . Assuming that the shape of the form-factors  $f$  are well approximated by simple pole forms and taking the pole mass for  $f^{(D \rightarrow K^*)}$  to be 2.5 GeV (corresponding to the  $D_s^{**}$  mass) and the pole mass for  $f^{(D \rightarrow \rho)}$  to be 2.4 GeV (corresponding to the  $D^{**}$  mass), we find that the ratio of  $D$  decay form-factors squared in eq. (23) varies by less than 0.5% over the range  $1 < y < 1.5$ . It may be possible to get some model-independent information on the  $y$ -dependence of the ratio  $f^{(D \rightarrow \rho)}/f^{(D \rightarrow K^*)}$  using the methods of Ref. [19].

The  $D$  semileptonic decay rate is almost completely saturated by the  $K$  and  $K^*$  hadronic final states. The heavy quark symmetry relations in eqs. (5) and (6) do not imply that the rare decay mode  $B \rightarrow X_s \nu \bar{\nu}$  (and also  $B \rightarrow X_s \ell \bar{\ell}$  when the effects of the four-quark



operators are neglected) is also saturated by these states in the kinematic region that overlaps with the  $D$  decay. For some of the  $D$  decay phase-space  $q^2$  is small compared with  $m_D^2$ , while the scaling relations in eqs. (5) and (6) hold for  $c$  and  $b$  quark masses treated as large with  $y$  held fixed.

#### IV. CONCLUDING REMARKS

In this paper we have explored the use of exclusive  $B$  and  $D$  decays to obtain a model-independent value of  $|V_{ub}|$  with small theoretical uncertainties. Our method is based on the fact that the Grinstein-type double ratio of form-factors  $(f^{(B \rightarrow \rho)}/f^{(B \rightarrow K^*)})/(f^{(D \rightarrow \rho)}/f^{(D \rightarrow K^*)})$  is equal to unity in the  $SU(3)$  limit, and in the limit of heavy quark symmetry. A determination of  $|V_{ub}|$  with an uncertainty from theory that is less than 10% is possible using information obtainable from the decay modes  $B \rightarrow \rho \ell \bar{\nu}_\ell$ ,  $B \rightarrow K^* \nu \bar{\nu}$ ,  $D \rightarrow \rho \bar{\ell} \nu_\ell$  and  $D \rightarrow K^* \bar{\ell} \nu_\ell$ . If, for  $1 < y < 1.5$ ,  $f^{(D \rightarrow \rho)}(y)/f^{(D \rightarrow K^*)}(y)$  is almost independent of  $y$  then a precise value for  $|V_{ub}|$  can be extracted from the rates for  $B \rightarrow \rho \ell \bar{\nu}_\ell$  and  $B \rightarrow K^* \nu \bar{\nu}$  integrated over this region in  $y$  (and  $f^{(D \rightarrow \rho)}(1)/f^{(D \rightarrow K^*)}(1)$ ). In a simple pole model this ratio of  $D$  decay form-factors is almost independent of  $y$ . We found that the matrix elements of the four-quark operators in the effective Hamiltonian for  $B \rightarrow K^* \ell \bar{\ell}$  induce about a 20% uncertainty for the  $B \rightarrow K^* \ell \bar{\ell}$  decay rate from  $c\bar{c}$  threshold effects in the region  $1 < y < 1.5$ .

At the present time the rare decays  $B \rightarrow K^* \nu \bar{\nu}$  and  $B \rightarrow K^* \ell \bar{\ell}$  have not been observed, and there is no information on the individual form-factors for  $D \rightarrow \rho \bar{\ell} \nu_\ell$ . Because of this, it is difficult to give a prognosis for the ultimate utility of the ideas presented here. However, even in the absence of the complete set of information needed for a high precision determination of  $|V_{ub}|$ , our results may be useful. CLEO has reported a yield of about 1000  $B \rightarrow \rho \ell \bar{\nu}_\ell$  events, corresponding to the branching ratio  $\text{Br}(\bar{B}^0 \rightarrow \rho^+ \ell \bar{\nu}_\ell) \simeq (2 - 3) \times 10^{-4}$  [20]. If heavy quark symmetry and  $SU(3)$  are employed to get  $|f^{(B \rightarrow \rho)}|^2 (1 + \delta^{(B \rightarrow \rho)})$  from the measured  $D \rightarrow K^* \bar{\ell} \nu_\ell$  form-factors, then eq. (3) can be used to extract  $|V_{ub}|$  from the

large  $q^2$  region of the Dalitz plot for the exclusive decay  $B \rightarrow \rho \ell \bar{\nu}_\ell$ . We predict, with this technique, a branching ratio of  $5.2|V_{ub}|^2$  for  $\bar{B}^0 \rightarrow \rho^+ \ell \bar{\nu}_\ell$  in the region  $1 < y < 1.5$ . Lattice Monte Carlo simulations [15] (and constituent quark model calculations [21]) suggest that the violations of heavy quark symmetry and  $SU(3)$  symmetry that give corrections to the relation between  $f^{(B \rightarrow \rho)}$  and  $f^{(D \rightarrow K^*)}$  are not anomalously large. This method will give a value for  $|V_{ub}|$  that is on a more sound theoretical footing than that which results from a comparison of the endpoint of the electron spectrum of inclusive semileptonic  $B$  decay with phenomenological models.

If experimental data on  $B \rightarrow K^* \nu \bar{\nu}$  is available before a detailed study of semileptonic form-factors for  $D \rightarrow \rho \bar{\ell} \nu_\ell$  is performed, then using eq. (22) an extraction of  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  should be possible. This gives a prediction for  $|f^{(B \rightarrow \rho)}|^2 (1 + \delta^{(B \rightarrow \rho)})$  with corrections of order  $m_s$ , but no order  $1/m_c$  corrections since heavy quark symmetry is not used. In this case there is no reason to restrict our analysis to the region of phase-space  $1 < y < 1.5$ . Lattice QCD results suggest that the influence of  $SU(3)$  violation on the form-factors is small, and hence the value of  $|V_{ub}|$  that can be extracted in this way will be fairly precise. A sizable uncertainty in the theoretical prediction for the  $B \rightarrow K^* \ell \bar{\ell}$  decay rate arises from the charmonium resonance region. Without a better understanding of this, it will not be possible to extract  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  from this decay mode with high accuracy. Nonetheless, an extraction of  $|f^{(B \rightarrow K^*)}|^2 (1 + \delta^{(B \rightarrow K^*)})$  from this mode may provide a useful determination of  $|f^{(B \rightarrow \rho)}|^2 (1 + \delta^{(B \rightarrow \rho)})$  (and hence  $|V_{ub}|$ ) with uncertainties now from both  $SU(3)$  violation and from the contribution of the four-quark operators to the  $B \rightarrow K^* \ell \bar{\ell}$  rate.

Some improvements on the analysis in this paper are possible. Combining chiral perturbation theory for mesons containing a heavy quark with heavy vector-meson chiral perturbation theory allows a computation of the order  $m_s \ln m_s$   $SU(3)$  violation in  $f$  [22]. Unfortunately such an analysis cannot give a definitive result on the size of the  $SU(3)$  violations because of unknown order  $m_s$  counterterms. In this paper we have neglected the lepton masses. It is possible to include the corrections that arise from the non-zero value of

the muon mass, although these are quite small.

A similar analysis to that performed in this paper can be done for the decays  $B \rightarrow \pi \ell \bar{\nu}_\ell$ ,  $B \rightarrow K \ell \bar{\ell}$ ,  $B \rightarrow K \nu \bar{\nu}$ ,  $D \rightarrow \pi \bar{\ell} \nu_\ell$  and  $D \rightarrow K \bar{\ell} \nu_\ell$ . However, in these decays there are complications because very near zero recoil “pole contributions” [23] spoil the simple scaling of the form-factors with the heavy quark mass.

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